Page No.:
Date: / /

Assignment = 03

Q1 Explain the Partial Differentiation with example

Ans Partial differentiation is used to differentiate mathematical function having more than one variable in them. In ordinary differentiation, we find derivative with respect to one variable only, as function contains only one variable.

Ex $f(x, y) = x^2y + y^2x$ with respect to x

$$\frac{\partial f}{\partial x} = 2xy + y^2$$

Now, with respect to y

$$\frac{\partial f}{\partial y} = x^2 + 2xy$$

Q2 If $u = \log(x^3 + y^3 + z^3 - 3xyz)$. Then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

Sol $u = \log(x^3 + y^3 + z^3 - 3xyz)$

$$\frac{\partial u}{\partial x} = \frac{1}{x^3 + y^3 + z^3 - 3xyz} \cdot (3x^2 - 3yz) \quad \text{—①}$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^3+y^3+z^3-3xyz}(3y^2-3xz) \quad -(2)$$

$$\frac{\partial u}{\partial z} = \frac{1}{x^3+y^3+z^3-3xyz}(3z^2-3xy) \quad -(3)$$

Adding (1), (2) & (3)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3x^2+3y^2+3z^2-3xy-3yz-3zx}{x^3+y^3+z^3-3xyz}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2-xy-yz-zx)}{x^3+y^3+z^3-3xyz}$$

$$\left[\because (a+b+c)(a^2+b^2+c^2-ab-bc-ac) = a^3+b^3+c^3-3abc \right]$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{(x+y+z)}$$

Hence Proove

Q3 If $u = \sin^{-1}\frac{x^2+y^2}{x+y}$. Then prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

Sol Given, $u = \sin^{-1} \frac{x^2+y^2}{x+y}$

$$\therefore \sin u = \frac{x^2+y^2}{x+y}$$

$$= \frac{x^2\left(1+\frac{y^2}{x^2}\right)}{x\left(1+\frac{y}{x}\right)}$$

$$= \frac{x\left(1+\frac{y^2}{x^2}\right)}{\left(1+\frac{y}{x}\right)}$$

$$= x f\left(\frac{y}{x}\right)$$

This is a homogeneous function of the degree 1.

So by Euler's Theorem

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

let, $z = \sin u$

$$\Rightarrow \quad x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

$$\Rightarrow \quad x\left(\frac{\partial}{\partial x}\right)\sin u + y\left(\frac{\partial}{\partial y}\right)\sin u = \sin u$$

$$\Rightarrow \quad x\cos u\left(\frac{\partial u}{\partial x}\right) + y\cos u\left(\frac{\partial u}{\partial x}\right) = \sin u$$

$$\Rightarrow \quad x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) = \frac{\sin u}{\cos u}$$

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

Hence prove

Page No.
Date: / /

Q4 If f is a homogeneous function of degree n, then

$$y\frac{d^2f}{dy^2} + x\frac{\partial^2 f}{\partial x \partial y} = (n-1)\frac{\partial f}{\partial y}$$

Sol By Euler's Theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf \quad -(1)$$

Partially differentiate (1) w.r.t. y

$$x\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial y^2} = n\frac{\partial f}{\partial y}$$

$$x\frac{\partial^2 f}{\partial x \partial y} + y\frac{\partial^2 f}{\partial y^2} = (n-1)\frac{\partial f}{\partial y}$$

$$\boxed{y\frac{\partial^2 f}{\partial y^2} + x\frac{\partial^2 f}{\partial x \partial y} = (n-1)\frac{\partial f}{\partial y}}$$

Hence proved.

Page No.
Date: / /

Q5 If f is a homogeneous function of degree n, Then

$$x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = n(n-1)f$$

Sol Given that f is homogenous of degree n.

Now, By Euler's Theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf \quad - (1)$$

Partially differentiate (1) w.r.t. x and y seperately.

$$x\frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial x} + y\frac{\partial^2 f}{\partial x \partial y} = n\frac{\partial f}{\partial x} \quad - (2)$$

$$x\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial y} + y\frac{\partial^2 f}{\partial y^2} = n\frac{\partial f}{\partial y} \quad - (3)$$

(2) * x + (3) * y

$$\Rightarrow x^2\frac{d^2 v}{dx^2} + 2xy\frac{\partial^2 v}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} = n(n-1)f$$

Hence Prove